LEVEL II

AD A108823

SYMMETRIC COMPLEMENTATION*

John H. Reif

TR-07-81

*This is a revised version of "O(log N) algorithms in sequential space and parallel time (for min spanning trees, k-connectivity, planarity and other symmetric games."

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A108823	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Symmetric Complementation	5. TYPE OF REPORT & PERIOD COVERED Technical Report
	6. PERFORMING ORG. REPORT NUMBER TR-07-81
7. AUTHOR(s) John H. Reif	8. CONTRACT OR GRANT NUMBER(s) N00014-80-C-0647
9. PERFORMING ORGANIZATION NAME AND ADDRESS Harvard University Cambridge, MA 02138	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research 800 North Quincy Street Arlington, VA 22217	12. REPORT DATE 1981
	13. NUMBER OF PAGES 42
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) same as above	15. SECURITY CLASS. (of this report)
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

unlimited

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

unlimited

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

parallel algorithm, planarity, connectivity, symmetric computation, probabilistic algorithm, randomized algorithm.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

See reverse

81 12 22 110

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20.

Summary. This paper introduces a class of 1 player games of perfect information, which we call *complementing games*; the player is allowed moves which complement the value of successive plays. A complementing game is *symmetric* if all noncomplement moves are reversible (i.e., form a symmetric relation). These games are naturally related to a class of machines we call *symmetric complementing machines*. Symmetric nondeterministic machines were studied in [Lewis and Papadimitriou, 80]; they are identical to our symmetric complementing machines with complement moves allowed only on termination. (A companion paper to appear will describe the computational complexity of symmetric complementing and alternating machines.) Of particular interest is the complexity class Σ_1^*CSYMLOG, which contains the outcome problem of symmetric complementing games with constant complement bound with game positions encoded in log space, and next move relations computable in log space. We show that the decision problem for a restricted quantified Boolean logic Σ_1^*QBF (sum) is complete in Σ_1^*CSYMLOG. We also show that Σ_1^*CSYMLOG contains many well-known and common combinatorial problems:

(1) minimum spanning forests

(2) k-connectivity and k-connected components

and also the recognition problems for many classes of graphs:

(3) planar graphs of valence 3

(4) chordal graphs

(5) comparability graphs

(6) interval graphs

(7) split graphs

(8) permutation graphs.

(cont. on next page)

20. Abstract (cont.)

We present a *probabilistic algorithm* (this is an algorithm which makes probabilistic choices [Rabin, 74], but with *no* assumptions about the probability distribution of the inputs) for recognizing the languages of Σ_*CSYMLOG within space $O(\log(n))$ and simultaneous time $n^{O(1)}$, with error probability $<\varepsilon$ for any given ε, $0 < \varepsilon < 1$. As a consequence, problems (1)-(8) can be done probabilistically in space $O(\log(n))$ and within simultaneous polynomial time. The best previous known algorithms for problems (1), (2) and (3) required deterministic space $\Omega(\log^2 n)$ [Ja'Ja' and Simon, 79], and algorithms for problems (4)-(8) previously required space $\Omega(n)$.

Also, we give a *probabilistic parallel algorithm* (which employs the Hardware Modification Machines of [Cook, 80], with probabilistic choice) for recognizing the languages of Σ_*CSYMLOG within parallel time $O(\log n)$ and error probability $<\varepsilon$, for any given ε, $0 < \varepsilon < 1$. Thus we also have parallel time $O(\log n)$ algorithms for problems (1)-(8). Our parallel algorithms seem practical since they require only a small polynomial number of processors. The best previously known parallel algorithms for problems (1)-(3) required parallel time $\Omega(\log^2 n)$ [Ja'Ja' and Simon, 80] and we know of no previous parallel algorithms for problems (4)-(8). Furthermore, we show (by a nonconstructive technique) that for each input length $n \geq 0$, the probabilistic choice can be eliminated in both our sequential and parallel algorithms. This does not affect the efficiency of the algorithms, but makes our algorithms nonuniform (i.e., we have a different algorithm for each input length).

SYMMETRIC COMPLEMENTATION

by

John H. Reif*

Aiken Computation Laboratory
Harvard University
Cambridge, MA 02138

*This work was supported in part by the National Science Foundation Grant NSF-MCS79-21024, the Office of Naval Research Contract N00014-80-C-0647.

Summary. This paper introduces a class of 1 player games of perfect information, which we call *complementing games*; the player is allowed moves which complement the value of successive plays. A complementing game is *symmetric* if all noncomplement moves are reversible (i.e., form a symmetric relation). These games are naturally related to a class of machines we call *symmetric complementing machines*. Symmetric nondeterministic machines were studied in [Lewis and Papadimitriou, 80]; they are identical to our symmetric complementing machines with complement moves allowed only on termination. (A companion paper to appear will describe the computational complexity of symmetric complementing and alternating machines.) Of particular interest is the complexity class Σ_*CSYMLOG, which contains the outcome problem of symmetric complementing games with constant complement bound with game positions encoded in log space, and next move relations computable in log space. We show that the decision problem for a restricted quantified Boolean logic Σ_*QBF$\oplus$ is complete in Σ_*CSYMLOG. We also show that Σ_*CSYMLOG contains many well-known and common combinatorial problems:

(1) minimum spanning forests

(2) k-connectivity and k-connected components

and also the recognition problems for many classes of graphs:

(3) planar graphs of valence 3

(4) chordal graphs

(5) comparability graphs

(6) interval graphs

(7) split graphs

(8) permutation graphs.

We present a *probabilistic algorithm* (this is an algorithm which makes probabilistic choices [Rabin, 74], but with *no* assumptions about the probability distribution of the inputs) for recognizing the languages of Σ_*CSYMLOG within space $O(\log(n))$ and simultaneous time $n^{O(1)}$, with error probability $<\varepsilon$ for any given $\varepsilon, 0<\varepsilon<1$. As a consequence, problems (1)-(8) can be done probabilistically in space $O(\log(n))$ and within simultaneous polynomial time. The best previous known algorithms for problems (1), (2) and (3) required deterministic space $\Omega(\log^2 n)$ [Ja'Ja' and Simon, 79], and algorithms for problems (4)-(8) previously required space $\Omega(n)$.

Also, we give a *probabilistic parallel algorithm* (which employs the Hardware Modification Machines of [Cook, 80], with probabilistic choice) for recognizing the languages of Σ_*CSYMLOG within parallel time $O(\log n)$ and error probability $<\varepsilon$, for any given $\varepsilon, 0<\varepsilon<1$. Thus we also have parallel time $O(\log n)$ algorithms for problems (1)-(8). Our parallel algorithms seem practical since they require only a small polynomial number of processors. The best previously known parallel algorithms for problems (1)-(3) required parallel time $\Omega(\log^2 n)$ [Ja'Ja' and Simon, 80] and we know of no previous parallel algorithms for problems (4)-(8). Furthermore, we show (by a nonconstructive technique) that for each input length $n>0$, the probabilistic choice can be eliminated in both our sequential and parallel algorithms. This does not affect the efficiency of the algorithms, but makes our algorithms nonuniform (i.e., we have a different algorithm for each input length).

1. INTRODUCTION

In the previous decade, considerable success has been made in the design of time optimal sequential algorithms for many combinatorial problems on graphs (such as spanning trees, k-connectivity for $k = 1,2$, and 3 [Hopcroft and Tarjan, 73b] and planarity testing [Hopcroft and Tarjan, 73a]) using the technique of *depth-first search*. Also, breadth-first search has been used for optimal algorithms for other graph problems. By applying well-known simulation results (for example, [Fortune and Wyllie, 78]), we can derive parallel space optimal algorithms from these sequential time optimal algorithms. Also, parallel time has been related to sequential space (by the simulation results of [Fortune and Wyllie, 78] and [Dymond and Cook, 80], for example). It is intriguing therefore to ask:

(i) Is there a general graph search technique which yields optimal sequential space algorithms and (either by simulation results or directly) also yields optimal parallel time algorithms?

We require that these algorithms be *reasonable*: that a sequential algorithm with space bound $S(n)$ uses no more than $2^{O(S(n))}$ sequential time (note that if $S(n) = O(\log n)$ then $n^{O(1)}$ sequential time will be used) and that a parallel algorithm with time bound $T(n)$ use no more than $2^{O(T(n))}$ processors (again, note that $T(n) = O(\log n)$ implies $n^{O(1)}$ processors will be used). (Note that certain probabilistic TMs have a time bound doubly exponential in these space bound and are there for not reasonable).

The depth-first search and breadth first search techniques appear not to be applicable to (i), due to their sequential nature. Another possible technique for solving a graph problem is to efficiently reduce the problem to boolean transitive closure, for which there is a known $\Omega(\log^2 n)$ parallel time algorithm [Csanky, 76]. [Ja'Ja' and Simon, 80] apply this

technique to do planarity testing and to solve connectivity problems in $\Omega(\log^2 n)$ parallel time. (However, boolean transitive closure has no known algorithm with less than $\Omega(\log^2 n)$ parallel time bound, and this bound seems very difficult to improve, whereas we show here the planarity testing and connectivity problems actually have $O(\log n)$ probabilistic parallel time algorithms.)

A related problem is:

(ii) Is there a logic, in which a significant class of combinatorial problems may be succinctly expressed, and such that validity of sentences in the logic may be decided efficiently or even optimally? (with respect to sequential space or parallel time)

A logic satisfying the conditions of (ii) could be used as the kernel of a language for parallel programming, where programs may be "compiled" into time optimal code for parallel machines.

This paper proposes solutions to (i) and (ii). A space efficient sequential *probabilistic search* technique was first introduced by [Aleliunas, Karp, Lipton, Lovász, and Rackoff, 79] to test 1-connectivity. We generalize the probabilistic search technique to yield optimal algorithms (in sequential space and also parallel time) for the complexity class Σ_*CSYMLOG, which contains a large class of many important combinatorial problems. Furthermore, we propose a restricted quantified boolean logic Σ_*QBF$\oplus$ as a solution to (ii).

This paper is organized as follows:

Section 2 defines complementing games and machines, complexity notation. Section 3 provides a sequential decision algorithm for the problems of Σ_*CSYMLOG, which runs in probabilistic space $O(\log n)$, and can be modified to run in nonuniform deterministic space $O(\log n)$.

Section 4 gives a probabilistic parallel algorithm which can simulate any sequential space $S(n)$ bounded probabilistic computation within parallel time $O(S(n))$. We also show in Section 4 that we can eliminate probabilistic choices in our parallel algorithm, without degrading its running time, but introducing nonuniformity. Section 5 introduces our logic Σ_*QBF$\oplus$ and shows its invalidity problem is complete in Σ_*CSYMLOG. We also show in Section 5 that various combinatorial problems (including minimum spanning trees, k-connectivity and graph recognition problems such as valence 3 planarity testing) are in Σ_*CSYMLOG.

2. PRELIMINARY DEFINITIONS

2.1 Symmetric Relations

Let $R \subseteq D \times D$ be a relation on domain D. Let its inverse be $R^- = \{(b,a) \mid aR\ b\}$. R is *symmetric* if $R = R^-$. R is *deterministic* if for all $a \in D$ there is at most one $b \in D$ such that aRb.

2.2 Complementing Games

A (1 player) *complementing game* consists of a quadruple $G = (P, W, \vdash, \vdash_c)$ where:

(i) P is the set of *positions*; P is assumed to be a set of strings over a finite alphabet.

(ii) $W \subseteq P$ are the *winning positions*,

(iii) $\vdash \subseteq P \times P$ is the *next move relation*, and

(iv) $\vdash_c \subseteq \vdash$ are the *complement moves*.

Fix an *initial position* $p_0 \in P$. A *move* is a pair $(p,p') \in \vdash$. The move (p,p') is an *initial move* (*termination move, complement move*) if $p = p_0$ (if p' has no next move, and $(p,p') \in \vdash_c$, respectively). We assume there is no next move from any winning position.

Let complement game G be *symmetric* if $\vdash_s = \vdash - \vdash_c$ is a symmetric relation (see 2.1). Let $\vdash$ be *log space* if there is a deterministic log space *next move transducer* which given any $p \in P$, outputs $\{p' | p \vdash p'\}$.

Let a *play* be a maximal length sequence of positions $p_0, p_1, \ldots$ where p_0 is the initial position, and $p_{i-1} \vdash p_i$ for $i = 1,2,\ldots$ (we allow a trivial play p_0). For any finite $k \geq 0$, let p_0 have *complement bound* k if any play from p_0 has $<k$ complements ignoring the initial move (ignoring the initial move allows us to maintain the duality between Σ_k and π_k).

Suppose p_0 has finite complement bound k. Let OUTCOME(p_0) = true if there exists a finite play prefix $p_0, p_1, \ldots, p_j$, $j \geq 0$, with no complement moves and where either $p_j \in W$ or there exists at least one complement move from p_j and OUTCOME(p') = false for each complement move $(p_j,p') \in \vdash_c$. Otherwise OUTCOME(p_0) = false. The *outcome problem* for G is given any $p \in P$, next move transducer for $\vdash$, and a recognizer for W, compute OUTCOME(p).

2.2 Machine Definitions*

Let a *complementing (Turing) machine* be a 9-tuple $M = (\Sigma, \Gamma, b, t, Q, q_0, Q_A, \delta, \delta_c)$ where

*See note at end of this paper on previous and equivalent machine definitions for symmetric alternating machines.

Σ is the finite *input alphabet*

Γ is the finite *tape alphabet*, with $\Sigma \subseteq \Gamma$

$b \in \Gamma$ is the distinguished *blank symbol*,

t is the *number of tapes* where tape 1 is the input tape

Q is the finite *state set*

$q_0 \in Q$ is the *initial state*

$Q_A \subseteq Q$ are the *accepting states*

$\delta \subseteq (Q \times (\Sigma^2 \times \{\underline{\text{left}},\underline{\text{right}}\}) \times (\Sigma^2 \times \{\underline{\text{left}},\underline{\text{right}}\})^{t-1})^2$ is the *transition relation*

$\delta_c \subseteq \delta$ are the *complement transitions*.

δ has a slightly nonstandard definition so that we can syntactically define symmetric machines in a manner similar to [Lewis and Papadimitriou, 80]. Suppose transition $((q,a_1b_1m_1,\ldots,a_tb_tm_t), (q',a_1'b_1'm_1',\ldots,a_t'b_t'm_t') \in \delta$ is taken. Then the previous state was q and the new state is q'. Each tape $i \in \{1,\ldots,t\}$ moves its head one cell in direction m_i', and m_i is the reverse of direction m_i'. If $m_i' = \underline{\text{right}}$ then previously the head of tape i was scanning symbol a_i and "peeking" at symbol b_i located one cell to the right; in the new configuration these symbols a_ib_i replaced by symbols $a_i'b_i'$ and the head is scanning symbol b_i'. The case $m_i' = \underline{\text{left}}$ is similar, except the head was previously scanning over symbol b_i while "peeking" at symbol a_i located one cell to the left; afterwards the head is scanning symbol a_i'. We assume there is no transition from an accepting state. Let M be *symmetric* if noncomplement transitions $\delta_s = \delta - \delta_c$ are a symmetric relation. Let $\mathcal{I}$ be the *configurations* of M, defined in the usual way for Turing machines. We may extend δ in the usual way to the *next move relation* $\vdash \subseteq \mathcal{I} \times \mathcal{I}$. Let $\vdash_c \subseteq \vdash$ be the next moves which are complements. Note that if M is symmetric then the relation $\vdash_s = \vdash - \vdash_c$

is symmetric. Let $W \subseteq \mathcal{I}$ be the accepting configurations. $G_M = (\mathcal{I}, W, \vdash, \vdash_c)$ is the *computation complementing game* of M.

Given an input string $\omega \in \Sigma^n$, we let the initial configuration $I_0(\omega)$ be the initial position of G_M. *Computation sequences* of M are plays of G_M from $I_0(\omega)$. Suppose M has a finite complement bound from $I_0(\omega)$. M *accepts* ω if OUTCOME$(I_0(\omega))$ = true. If M has only complement moves which are terminations, than M is a *nondeterministic machine*; furthermore, if $\vdash_s$ is symmetric than M is a *symmetric nondeterministic machine* as defined by [Lewis and Papadimitriou, 80]. Let $L(M) = \{\omega \in \Sigma^* | M \text{ accepts } \omega\}$.

2.3 Complexity Classes

Let complementing machine M have *space bound* S(n) (complement *bound* K(n)) if on any input of length $n \geq 0$, each computation sequence has no more than S(n) nonblank cells on any work tape in each configuration (less than K(n) complements on any computation sequence ignoring the initial moves).

Note that nondeterministic and co-nondeterministic machines have complement bound 1 (see 2.2). For notational simplicity, we define a complementing machine with complement bound 0 to be a deterministic TM. Let $\mathcal{M}$ be a class of complementing machines. Let $\mathcal{M}$SPACE(S(n)) be the languages accepted by those machines in $\mathcal{M}$ with space bound S(n). Let $\Sigma_k\mathcal{M}$SPACE(S(n)) ($\Pi_k\mathcal{M}$SPACE(S(n))) be the languages accepted by those machines in $\mathcal{M}$ with space bound S(n), complement bound k and no complement moves (only complement moves, respectively) for the initial moves. Let $\Sigma_*\mathcal{M}$SPACE(S(n)) = $\bigcup_{k\geq 0} \Sigma_k\mathcal{M}$SPACE(S(n)); that is the machines operate in some constant number of complements, independent of the input length.

In the context of complexity classes, we let D denote the class of deterministic TMs, let N denote the nondeterministic TMs, and let NSYM be the symmetric nondeterministic machines. Let C be the complementing machines and let CSYM be the symmetric complementing machines. For example, the complexity class NSYMSPACE(S(n)) = {L|L is accepted by a symmetric nondeterministic machine with space S(n)} previously investigated by [Lewis and Papadimitriou, 80]. The complexity class $\Sigma_{K(n)}$CSYMSPACE(S(n)) = {L|L is accepted by a symmetric complementing machine with space bound S(n), complement bound K(n), and no complement initial moves} is of central importance to this paper. For notational simplicity, let NSYMLOG = NSYMSPACE(log(n)), and CSYMLOG = CSYMSPACE(log(n)).

Let $L_1 \leq_{\log} L_2$ denote that language L_1 can be many-one reduced in deterministic log-space to language L_2. Let L_1 be $\leq_{\log}$ *equivalent* to L_2 if $L_1 \leq_{\log} L_2$ and $L_2 \leq_{\log} L_1$. Let L_2 be *complete* in a family of languages $\mathcal{L}$ if $L_2 \in \mathcal{L}$ and $L_1 \leq_{\log} L_2$ for each $L_1 \in \mathcal{L}$. Note that if $S(n) \geq \log n$, $L_1 \leq_{\log} L_2$, and $L_2 \in$ CSYMSPACE(S(n)), then $L_1 \in$ CSYMSPACE(S(n)), by Proposition 2.3.

2.4 Preliminary Results for Symmetric Complementing Machines

It follows immediately from our definition of complementing machines that:

PROPOSITION 2.1. For any $L \subseteq \Sigma^*$, $L \in \Sigma_{K(n)}$ CSYMSPACE(S(n)) iff $\Sigma^* - L \in \Pi_{K(n)}$ CSYMSPACE(S(n)).

Since any complementing machine has a finite tape alphabet, it is easy to show

PROPOSITION 2.2. If complementing machine M has space bound $S(n) \geq \log n$, then M has complement bound $2^{O(S(n))}$.

[Lewis and Papadimitriou, 80] show DSPACE(S(n)) $\subseteq$ NSYMSPACE(S(n)). Also, since any symmetric complementing machine is a complementing machine:

PROPOSITION 2.3. DSPACE(S(n)) $\subseteq \Sigma_{K(n)}$ CSYMSPACE(S(n)) $\subseteq \Sigma_{K(n)}$ CSPACE(S(n)).

(Note that space bounded complementing machines accept the same languages as space bounded alternating machines and that complementing machines without space bounds accept some not r.e. languages but that is not relevant to this paper. Further properties of symmetric complementing and alternating machines will appear in a companion paper, but are not required for the results of this paper.)

3. A SPACE EFFICIENT DECISION ALGORITHM FOR SYMMETRIC COMPLEMENTING MACHINES

We give a O(S(n)K(n)) space sequential algorithm for recognizing the languages of $\Sigma_{K(n)}$ CSYMSPACE(S(n)). The algorithm is probabilistic (see 3.1 and 3.2), though we show it can be made deterministic by introducing nonuniformity (see 3.3).

3.1 Probabilistic Sequential Machines

We define a *probabilistic* TM to be a multitape deterministic Turing Machine PM with a special read-only, one-way tape (distinct from the input and work tapes) containing an infinite binary sequence. The contents of this "random bitvector" tape are chosen randomly on each execution of PM. Let Σ be the input alphabet of PM and let $L \subseteq \Sigma^*$. For any $\varepsilon(n)$, $0 \leq \varepsilon(n) < 1$ say PM *recognizes* L *within error* $\varepsilon(n)$ if for all $\omega \in \Sigma^n$,

C1: $\omega \in L$ implies Prob(PM accepts ω) $\geq 1 - \varepsilon(n)$.

C2: $\omega \notin L$ implies Prob(PM accepts ω) $< \varepsilon(n)$.

To justify this definition, we note that Adleman's [78] definition of acceptance of probabilistic machines is similar to ours, except $\varepsilon(n) = 1/2$ in C1 and he strengthens condition C2 by requiring that $\omega \notin L$ imply PM does not accept ω on any probabilistic choice. Many probabilistic algorithms in number theory satisfy this more restrictive property, but it is too restrictive for many of the applications in this paper. On the other hand, Gill [77] defines acceptance of probabilistic machines with the max of the error of acceptance and rejection less than 1/2.

Note that a probabilistic machine may not be *reasonable* in the sense defined in the introduction (since Gill [77] gives a probabilistic machine with space bound $S(n)$ and expected time bound $2^{2^{O(S(n))}}$); however, the probabilistic machine implementing the PROB-SEARCH algorithm of Section 3.2 will be reasonable.

3.2 Probabilistic Simulation of Space Bounded Symmetric Complementing Machines

We shall show:

THEOREM 3.1. For any $\varepsilon(n)$, $0 < \varepsilon(n) < 1$, there is a probabilistic TM which recognizes $L \in \Sigma_{K(n)}\text{CSYMSPACE}(S(n))$ within given error $\varepsilon(n)$ and space $O(K(n)(S(n) + \log d(n)))$ and time $(d(n)2^{O(S(n))})^{K(n)}$, where $d(n) = K(n)\,(O(S(n)) + \log(O(K(n))) - \log \varepsilon(n)$.

Note that if $\varepsilon(n)$ is constant, then by Proposition 2.2, we require space $O(K(n)S(n))$ and time $2^{O(K(n)S(n))}$. Thus:

COROLLARY 3.1. For each constant ε, $0 < \varepsilon < 1$, and $L \in \Sigma_*$ CSYMLOG, there is a probabilistic TM which recognizes L within error ε and simultaneous space $O(\log n)$ and time $n^{O(1)}$.

Although Theorem 3.1 suffices for our applications, we also show its space bounds can be improved.

THEOREM 3.2. For each $L \in \Sigma_{K(n)}$ CSYMSPACE(S(n)), there is a probabilistic TM which recognizes L within error $\varepsilon(n)$ and space $O(K(n)(S(n) + \log(S(n) + \log d(n))))$.

Our probabilistic search technique will utilize the following result:

LEMMA 3.1. [Aleliunas, Karp, Lipton, Lovasz, and Rackoff, 79]. Let $G = (V,E)$ be any undirected, connected graph. Let r be a *random walk* in G from any vertex $v \in V$ be constructed from trivial path v by repeatedly extending the front end of r by adding a random edge of E which is connected to the current front end vertex of r. Let r be a random walk of length $2|E|(|V| - 1)$. Then Prob(r visits all vertices in V) $\geq 1/2$.

[Lewis and Papadimitriou, 80] observe that this Lemma immediately implies a space $O(S(n))$ probabilistic algorithm for NSYMSPACE(S(n)). A generalized probabilistic search technique is used here to decide acceptance of symmetric complementing machines.

We now prove here Theorems 3.1 and 3.2. Let M be a symmetric complementing machine as defined in Section 2.2. We assume M has complement bound $K(n) \geq 1$ and constructible space bound $S(n) \geq \log n$ (otherwise, we use the standard technique of trying $S(n) = \log n, 1 + \log n, \ldots$ to the

construction given below). Let $\mathcal{I}$ be the set of configuration of M and let $W \subseteq \mathcal{I}$ be the accepting configurations. Let $\vdash$ be the next move relation of M. Let $\vdash_c, \vdash_s \subseteq \vdash$ be the complement and noncomplement moves of $\vdash$, respectively. Fix some $n \geq 0$. Let $\mathcal{I}' \subseteq \mathcal{I}$ be the configurations which have $\leq S(n)$ nonblank cells on each work tape.

We define a recursive procedure which takes as input a configuration $I \in \mathcal{I}'$. Also, the procedure has a global variable t (which determines the procedure's probability of success).

```
procedure PROB-SEARCH_t(I)
   begin
      local integer i, set COMP
      i ← 0
      while i ≤ t do
         begin
            if I is accepting then return true
            COMP ← {I' ∈ I' | I ⊢_c I'}
            if COMP ≠ ∅ then
               if PROB-SEARCH(I') = false for all I' ∈ COMP then
                                                  return true
            choose a random I' from {I' ∈ I' | I ⊢_s I'}
            I ← I'
            i ← i + 1
         end
      return false
   end
```

For each k, $1 \leq k \leq K(n)$ let $\mathcal{I}_k \subseteq \mathcal{I}'$ be the configurations which also have complement bound k.

Let $\varepsilon_{k,t}$ be the max probability that PROB-SEARCH$_t$(I) = false for any $I \in \mathcal{I}_k$ such that OUTCOME(I) = true. Let $\bar{\varepsilon}_{k,t}$ be the max probability

that PROB-SEARCH$_t$(I) = true for any $I \in \mathcal{I}_k$ such that OUTCOME(I) = false. Thus, $\varepsilon_{k,t}$ and $\bar{\varepsilon}_{k,t}$ are the worst case error probabilities for rejection and acceptance, respectively.

LEMMA 3.2. There are constants $b,c \geq 1$, dependent only on M, such that for any $d \geq 1$, $k \geq 1$,

$$\max(\varepsilon_{k,t}, \bar{\varepsilon}_{k,t}) \leq k2^{-d}(tb)^{k-1}$$

where $t = 2dc^{S(n)}$.

Proof. Let $\vdash_s' = (\mathcal{I}' \times \mathcal{I}') \cap \vdash_s$. Since M is symmetric $(\mathcal{I}', \vdash_s')$ is an undirected graph. There exists a constant $b \geq 1$ upper bounding the number of next configurations $\{I' | I \vdash I'\}$ possible from any given configuration I. Also there exists a constant $c_1 \geq 1$ such that $|\mathcal{I}'| \leq c_1^{S(n)}$. Thus there exists a constant $c_2 \geq 0$ such that $|\vdash_s'| \leq c_2^{S(n)}$. Let $c = c_1 \cdot c_2$ so $t \geq 2d|\mathcal{I}'|\,|\vdash_s'|$.

For each pair of configurations $I, I' \in \mathcal{I}_k$ for which there is a non-complementing computation sequence from I to I', by Lemma 3.1, we have prob(r visits I') $\geq 1 - 2^{-d}$ for a random walk r in $(\mathcal{I}_k, \vdash_s')$ starting at I and of length $2d|\mathcal{I}'|\,|\vdash_s'|$.

Now we prove Lemma 3.2 by induction on k. For $k = 1$ we show $\max(\varepsilon_{1,t}, \bar{\varepsilon}_{1,t}) \leq 2^{-d}$. The worst case error probability for rejection and acceptance from any $I \in \mathcal{I}_1$ with no complement next move is $\leq 2^{-d}$ and 0, respectively. Thus the total worst case error probability for rejection and acceptance from any $I \in \mathcal{I}_1$ is $\leq 2^{-d}$ and $\leq 2^{-d}$, respectively.

Since there at most tb direct calls to PROB-SEARCH during a single execution of the body of the PROB-SEARCH procedure, for $k > 1$ we have:

$$\varepsilon_{k,t} \leq 2^{-d} + tb\bar{\varepsilon}_{k-1} \quad \text{and} \quad \bar{\varepsilon}_{k,t} \leq tb\varepsilon_{k-1} \quad .$$

By the induction hypothesis,

$$\max(\varepsilon_{k-1,t}, \bar{\varepsilon}_{k-1,t}) \leq (k-1)2^{-d}(tb)^{k-2} \quad .$$

Hence

$$\begin{aligned} \max(\varepsilon_{k,t}, \bar{\varepsilon}_{k,t}) &\leq 2^{-d} + tb\bar{\varepsilon}_{k-1} \\ &\leq 2^{-d} + (k-1)2^{-d}(tb)^{k-1} \\ &\leq k2^{-d}(tb)^{k-1} \quad . \end{aligned}$$

□

Let L be the language accepted by M. Suppose we are given some error function $\varepsilon(n)$, $0 < \varepsilon(n) < 1$. Let PM be the probabilistic TM which on input $\omega \in \Sigma^n$, computes $\text{PROB-SEARCH}_{t(n)}(I_0(\omega))$ and accepts iff the result is *true*, provided that $d(n) = K(n)\log(t(n)b) - \log \varepsilon(n)$ and $t(n) = 2d(n)c^{S(n)}$. (Note that both $d(n)$ and $t(n)$ are decreasing functions of $\varepsilon(n)$.) By Lemma 3.2, PM recognizes L within error $\varepsilon_{K(n)} \leq \varepsilon(n)$. Furthermore, PM has time bound $O(t(n))^{K(n)} = (d(n)2^{O(S(n))})^{K(n)}$. PM has space bound $O(K(n)(S(n) + \log d(n)))$ since we must store $b = O(1)$ configurations of size $O(S(n))$, and a "time counter" requiring space $\log t(n) = O(S(n) + \log d(n))$, to implement each of the $K(n)$ recursive cells. Thus we have proved Theorem 3.1. □

Although Theorem 3.1 is good enough for our applications to Σ_* CSYMLOG, it is nevertheless interesting to observe that we may decrease the space bound by using a trick due to Gill [77]. To avoid storing the "time counter" in the procedure PROB-SEARCH_t, we instead sample a random bit on each iteration of the while statement. If at any time there have been

log t consecutive zero's chosen, then we immediately exit the while statement. This test replaces the test $(i \leq t)$ in the original text of PROB-SEARCH_t. To achieve error $\varepsilon_{K(n)} \leq \varepsilon(n)$, we must only increase $d(n)$ by a factor of $1/(1-\log \exp(1))$. Only $O(K(n)(S(n) + \text{loglog } t(n))) = O(K(n)(S(n) + \log(S(n) + \log d(n))))$ space is required by this method (but note that we no longer have a time bound . Thus we have proved Theorem 3.2. □

3.3 Eliminating Probabilistic Choices

Let a *nonuniform deterministic* TM be a deterministic TM augmented with a special read-only tape, called the *advice tape*, whose contents are fixed for all inputs of the same length n, but which may have different contents for distinct input lengths n and n'. (Neither the input tape nor the advice tape is considered in the space bound of this machine.) This nonuniform machine has *advice bound* A(n) if on inputs of length n, the advice tape has A(n) cells (see [Karp and Lipton, 80]). We will show:

THEOREM 3.3. Each $L \in \Sigma_{\nu(n)}$ CSYMSPACE(S(n)) is accepted by a nonuniform deterministic TM within space bound O(K(n)S(n)), time bound $2^{O(K(n)S(n))}$, and advice bound $2^{O(S(n))}$.

COROLLARY 3.3. Each $L \in \Sigma_*$ CSYMLOG is accepted by a nonuniform deterministic TM within simultaneous space bound O(log n), time bound $n^{O(1)}$ and advice bound $n^{O(1)}$.

We now prove Theorem 3.3. We require a technical graph theoretic result.

Let $G = (V,E)$ be a undirected regular graph, with valence b. We assume G has a fixed adjacency list representation, so for each vertex v we have a list $\ell(v)$ of vertices adjacent to v. Given a string $U \in \{1,\ldots,b\}^*$, and a vertex v, let $U(G,v)$ be the path $v = v_0,\ldots,v_{|\sigma|}$ such that v_i is the $U(i)$ element of list $\ell(v_{i-1})$ for $i = 2,\ldots,|\sigma|$. Let $\mathcal{G}_{n,b}$ be the class of all undirected, regular graphs with $\leq n$ vertices and valence b. Let $U \in \{1,\ldots,b\}^*$ be (n,b)-*universal* if for each graph $G \in \mathcal{G}_{n,b}$ and each vertex v of G, $U(G,v)$ visits all the vertices of G.

LEMMA 3.3. [Aleliunas, Karp, Lipton, Lovasz, and Rackoff,79] For each $b \geq 1$, there is a $c(b)$ such that for each $n \geq 0$ there is a (n,b)-universal string $U_{n,b}$ of length $\leq c(b)n^3 \log n$.

Let M be a symmetric complementing machine of Section 3.2 with complement bound $K(n)$, and space bound $S(n)$. Let $(\mathcal{I}', \vdash_s')$ be the undirected graph of valence b defined in the Proof of Lemma 3.2. Clearly we can add redundant transitions so that $(\mathcal{I}', \vdash_s')$ is regular with valence b. Let NONUNIFORM-SEARCH$_t$(I) be the deterministic procedure derived from PROB-SEARCH$_t$(I) of Section 3.2 by using the $(|\mathcal{I}'|,b)$-universal string $U_{|\mathcal{I}'|,b}$ in place of probabilistic choice, for choosing the configurations to be explored in $(\mathcal{I}', \vdash_s')$.

By the Proof of Lemma 3.2, there exists some $c_1 \geq 1$ such that $|\mathcal{I}'| \leq c_1^{S(n)}$. Let $t(n) = c(b)(S(n)\log c_1)c_1^{3S(n)}$. Then Lemma 3.3 immediately implies

LEMMA 3.4. For each input string $\omega \in \Sigma^n$,

NONUNIFORM-SEARCH$_{t(n)}(I_0(\omega))$ = <u>true</u> iff M accepts ω.

This procedure may be implemented by a nonuniform deterministic TM with space bound $O(K(n)S(n))$, time bound $t(n)^{K(n)}=2^{O(K(n)S(n))}$, and advice bound $t(n)=2^{O(S(n))}$. Thus we have proved Theorem 3.3. □

4. A PARALLEL ALGORITHM

4.1 The Hardware Modification Machine

Our parallel machine model is the Hardware Modification Machine (HMM) of [Dymond and Cook, 80] (through we consider probabilistic and nonuniform variants of it below). The HMM was invented as the parallel analog of the storage modification machine of [Schönhage, 79]. The HMM seems to be the simplest possible parallel machine with modifiable storage structure, and the HMM can be simulated within real time, with the same number of processors, by many other such parallel machines, including the P-RAM of [Fortune and Wyllie, 78] (this P-RAM model was assumed for the parallel graph algorithms of [Ja'Ja' and Simon, 80]), the PRAM of [Savitch and Stimson, 79], and the SIMDAGs of [Goldschlager, 78].

Intuitively, a HMM consists of a finite collection of deterministic finite state machines which we call *processors*. The state transition function of these processors are identical. Each processor also contains the same fixed, finite number of *input* and *output connections* for transmission of values, from a finite alphabet, between processors. On each step (the state transitions of the processors are synchronous) a processor will read the values of its input connections which were set by its neighboring processors on the last step, and write new values on each of its output connections, (only one process is associated with each output

connection), and enter a new state. In addition, a processor may reconnect any input connection to any machine which can be reached by a path of length ≤ 2 from the previous input connection. Also, a processor may reconnect an input connection to a new processor (with the same finite state control, initialized in some given state and with input connections directed to its creator).

Given an input string $\omega \in \Sigma^n$, we assume the *initial configuration* of the HMM consists of a chain of $n+1$ identical processors $PP_0, PP_1, \ldots, PP_n$ each in the same initial state, and each with input connections connected back to itself, except each PP_{i-1} for $0 < i \leq n$, has a distinguished input connection to PP_i where the value output by PP_i is the i-th symbol of the input string ω. (This initialization scheme is somewhat simpler than that defined by [Dymond and Cook, 80], but yields the same technical results of interest here.) The HMM *accepts* ω if PP_0 ever enters a distinguished accepting state q_A.

The *time bound* $T(n)$ (*processor bound* $P(n)$) of the HMM is the maximum number of steps (processors, respectively) taken on any accepting computation for any input of length n. Generally we assume the HMM is *uniform*: the processors have the same finite state transition function for all input strings. However, we consider in Section 4.4 *nonuniform HMMs* which must only have the same finite state transitions for all input strings of the same length. The *advice bound* $A(n)$ of a nonuniform HMM is the number of tuples defining the processor state transition function for input of length n.

4.2 The Probabilistic HMM

In addition to the above of a uniform HMM, suppose we allow each processor PP_i *probabilistic choice* by providing a special read-only register r_i which is set randomly to 0 or 1 each step. Let PPM be the resulting *probabilistic* HMM. PMM recognizes *language* $L \subseteq \Sigma^*$ *within error* $\varepsilon(n)$, $0 \leq \varepsilon(n) < 1$, if for all $\omega \in \Sigma^n$,

C1: $\omega \in L$ implies Prob{PPM accepts ω} $\geq 1 - \varepsilon(n)$

C2: $\omega \notin L$ implies Prob{PPM accepts ω} $< \varepsilon(n)$.

(Note that the conditions C1, C2 for probabilistic recognition are identical to those given in 3.1. [Reif, 81] gives complexity bounds for various other probabilistic parallel machines and for both the [Adleman, 78] and also the [Gill, 77] definitions of probabilistic acceptance; if Adleman's definition of acceptance is used, than we can eliminate probabilistic choice in our parallel machines by introducing nonuniformity without any increase in parallel time; on the other hand if Gill's definition of probabilistic acceptance is used, then we show that probabilistic parallel space $S(n)$ contains parallel space $S(n)$ with nondeterministic choice.)

4.3 Parallel Simulation of Probabilistic Sequential Computations

[Dymond, and Cook, 80] prove that

THEOREM 4.1. If $L \in DSPACE(S(n))$ for $S(n) \geq \log n$, then L is recognized by a (deterministic) HMM in simultaneous parallel time bound $O(S(n))$ and processor bound $2^{O(S(n))}$.

We generalize their results to probabilistic computations.

THEOREM 4.2. Let PM be a probabilistic TM with space bound $S(n) \geq \log n$ and time bound $T(n)$. Suppose for some $\varepsilon(n)$, $0 < \varepsilon(n) < 1$, $L \subseteq \Sigma^*$ is recognized by PM within error $\varepsilon(n)$. Then there is a probabilistic HMM which recognizes L within error $\varepsilon(n)$ and with $O(S(n) + \log T(n))$ parallel time and utilizes $T(n) \cdot 2^{O(S(n))}$ processors. Furthermore, this HMM is uniform.

Proof. Fix some input string $\omega \in \Sigma^n$ and let $I_0(\omega)$ be the initial configuration of PM. Let $\mathcal{I}'$ be the configurations of PM using $\leq S(n)$ tape cells. Clearly there exists a constant $c>0$ such that $|\mathcal{I}'| \leq c^{S(n)}$. We assume $S(n)$ and $T(n)$, are constructible (otherwise we must in parallel use a diagonalization of $S(n) = 0,1,\ldots,$ and $T(n) = 0,1,\ldots$).

Our simulating probabilistic HMM, which we call PPM, will utilize a processor $PP_{I,t}$ for each t, $0 \leq t \leq T(n)$ and $I \in \mathcal{I}'$. These processors can be created in binary tree fashion within $O(\log(T(n)|\mathcal{I}'|))$ time. Each processor $PP_{I,t}$ chooses a configuration I' randomly from those allowed from configuration I by PM. $PP_{I,t}$ then makes a distinguished *jump* connection to processor $PP_{I',t+1}$. These connections can be made in time $O(\log(|\mathcal{I}'|))$, again using binary trees for indexing. Thereafter, each process $PP_{I,t}$ repeatedly connects its jump connection to that which was its jump connection of distance 2 in the previous step. These steps are executed synchronously by all the processes, and the HMM is allowed to halt and accept only when process $PP_{I_0(\omega),0}$ has a jump connection to a process $P_{I,t}$ where I is an accepting configuration of PM.

Suppose $I_0, I_1, \ldots$ is an execution sequence of M, with particular probabilistic choices r. Suppose also that the RAMs of PPM make particular probabilistic choices r', such that $PP_{I_t,t}$ initially sets its

jump connection to process $PP_{I_{t+1},t+1}$ for $t = 0,1,\ldots,T(n)-1$. Then it is easy to verify that PPM accepts ω (when making probabilistic choices r') iff PM accepts ω (when making probabilistic choices r). Since r and r' are chosen randomly, it follows that

$$\text{Prob}\{\text{PM accepts } \omega\} = \text{Prob}\{\text{PPM accepts } \omega\} .$$

Furthermore, if PPM accepts ω, then there is a path $PP_{I_0,0}, P_{I_1,1}, \ldots, P_{I_t,t}$ induced by the initial jump connections such that $I_0(\omega) = I_0, I_1, \ldots, I_t$ is an accepting computation of M, and $t \leq T(n)$. On each iteration, this path's length decreases by a factor of 1/2. Thus, PPM accepts within parallel time

$$O(\log(T(n)|\mathcal{I}'|)) = O(\log(T(n)2^{O(S(n))})) = O(S(n) + \log T(n)) . \quad \square$$

Combining Theorems 3.1 and 3.2, we have:

THEOREM 4.3. For any $S(n) \geq \log n$, $K(n) \geq 1$, and for each $\varepsilon(n)$, $0 < \varepsilon(n) < 1$ and $L \in \Sigma_{K(n)}\text{CSYMSPACE}(S(n))$, there is a probabilistic HMM which recognizes L within error $\varepsilon(n)$, with parallel time bound $O(K(n)(S(n) + \log d(n)))$ and processor bound $(d(n)2^{O(S(n))})^{K(n)}$ where $d(n)$ is defined as in Theorem 3.1.

Note that if $\varepsilon(n)$ is constant, then HMM has parallel time bound $O(K(n)S(n))$ and processor bound $2^{O(K(n)S(n))}$. Thus:

COROLLARY 4.3. For each constant ε, $0 < \varepsilon < 1$, and $L \in \Sigma_*\text{CSYMLOG}$, there is a probabilistic HMM which recognizes L within error ε and with parallel time $O(\log n)$ and $n^{O(1)}$ processor bound.

4.4 A Nonuniform Parallel Algorithm

By Theorems 4.1 and 3.3., we can eliminate probabilistic choice in our parallel algorithm, by introducing nonuniformity.

THEOREM 4.4. For each $L \in \Sigma_{K(n)}\text{CSYMSPACE}(S(n))$ with $S(n) \geq \log n$, L is accepted by a nonuniform HMM with parallel time bound $O(K(n)S(n))$, processor bound $2^{O(K(n)S(n))}$ and advice bound $2^{O(S(n))}$.

COROLLARY 4.4. Each $L \in \Sigma_*\text{CSYMLOG}$ is accepted by a nonuniform HMM within simultaneous parallel time $O(\log n)$, processor bound $n^{O(1)}$ and advice bound $n^{O(1)}$.

5. COMPUTATIONAL PROBLEMS IN Σ_*CSYMLOG

5.1 Symmetric Complementing Games

Let CSYMGAMES(S(n),K(n)) be the outcome problem for symmetric complementing games with positions of plays (starting at an initial position of size n) of length $\leq S(n)$ complement bound K(n), log space next move relation and log space recognizer for winning positions. By our definition of complementing machines, we have:

THEOREM 5.1. For $S(n) \geq \log n$, CSYMGAMES(S(n),K(n)) is complete for the languages accepted by symmetric complementing machines with space bound S(n), and complement bound K(n).

COROLLARY 5.1. The outcome problems

$\bigcup_{k\geq 0}$ CSYMGAMES(log n,k) are complete for Σ_*CSYMLOG.

5.2 QBF⊕

Given a set X of boolean variables, let literals$(X) = X \cup \{\neg x \mid x \in X\} \cup \{\underline{\text{true}}, \underline{\text{false}}\}$. Let CNF⊕ be the set of boolean formulas consisting of a conjunction of clauses, each clause consisting of the exclusive-or $\ell \oplus \ell'$ of two literals ℓ, ℓ'. Note that $\ell \oplus \ell'$ is equivalent to $(\neg\ell) \oplus (\neg\ell')$.

[Jones, Lien, and Laaser, 76] and [Lewis and Papadimitriou, 80] show CNF⊕ unsatisfiability is complete in NSYMLOG. Let Σ_0QBF⊕ and Π_0QBF⊕ be the truth values $\{\underline{\text{true}}, \underline{\text{false}}\}$. Inductively, let Σ_kQBF⊕ be the set of quantified boolean formulas F of the form $(\exists X) C_1 \wedge \ldots \wedge C_m$ where X is a set of boolean variables and each clause C_i is of either form $\ell \oplus \ell'$ or of the form $\ell \vee F'$ where $\ell, \ell' \in$ literals(X) and F' is a formula of Π_{k-1}QBF⊕. Also, let Π_kQBF⊕ be the set of quantified boolean formulas F of the form $(\forall X) C_1 \vee \ldots \vee C_m$ where each clause C_i is of the form $\ell \oplus \ell'$ or of the form $\ell \wedge F'$ where $\ell, \ell' \in$ literals(X) and formula F' must be in Σ_{k-1}QBF⊕. Let Σ_*QBF⊕ $= \bigcup_{k \geq 0} \Sigma_k$QBF⊕. Note that all variables are bound in QBF⊕ formulas.

THEOREM 5.2. For all $k \geq 0$, the invalidity problem for Σ_kQBF⊕ is complete in Σ_kCSYMLOG, and the invalidity problem for Π_kQBF⊕ is complete in Π_kCSYMLOG.

We prove here Theorem 5.2. Our proof requires a technical Lemma. This Lemma is an easy generalization of a result of [Jones, Lien, and Laaser, 76] which characterized decidability of CNF⊕ formulas.

LEMMA 5.1. Let F be a formula of Σ_kQBF⊕. F is invalid iff there exists a sequence of literals $\ell_0, \ldots, \ell_j$ such that $\ell_0 \oplus \neg\ell_1, \ldots, \ell_{j-1} \oplus \neg\ell_j$ are equivalent to clauses of F and $\ell_0 = \neg\ell_j$ or both (1) and (2) hold.

(1) $\ell_0 = \underline{\text{true}}$ or $\ell_0 \vee F'$ is a clause of F where F' is an invalid formula in $\Pi_{k-1}QBF\oplus$.

(2) $\ell_j = \underline{\text{false}}$ or $\neg\ell_j \vee F''$ is a clause of F where F'' is an invalid formula in $\Pi_{k-1}QBF\oplus$.

It will also be useful to note that

PROPOSITION 5.1. If F is a formula of $\Pi_k QBF\oplus$, then $\neg F$ is equivalent to a formula $\hat{F}$ of $\Sigma_k QBF\oplus$, where $\hat{F}$ is formed by switching the quantification symbols $\forall,\exists$ and also switching the logical connectives $\vee,\wedge$ in F. So F is invalid iff $\hat{F}$ is valid.

Proof of Theorem 5.2 by induction on k. $\Pi_0 QBF\oplus$ and $\Sigma_0 QBF\oplus$ can easily be shown complete in $\Pi_0 CSYMLOG = \Sigma_0 CSYMLOG = DSPACE(\log n)$.

Suppose for some $k \geq 1$ the Theorem holds for all $k' < k$. Let F be a $\Sigma_k QBF\oplus$ formula of length n. To decide F, we play a symmetric complementing game. Let the player begin by choosing a sequence of literals $\ell_0,\ldots,\ell_j$ such that $\ell_0 \oplus \neg\ell_1,\ldots,\ell_{j-1} \oplus \neg\ell_j$ are equivalent to clauses of F. Note that only the first literal and last literal need be stored, and this requires $O(\log n)$ space. This choice sequence is reversible since $(\ell_{i-1} \oplus \neg\ell_i) \equiv (\ell_i \oplus \neg\ell_{i-1})$. The player enters the accepting state (and thus wins) if either $\ell_0 = \neg\ell_j$ on both cases (1), (2) of Lemma 5.2 holds. This may require deciding formulas F', F'' of $\Pi_{k-1}QBF\oplus$. To do this, we allow the player two simultaneous complement moves from the current position. In these complement moves, we let the player test if both F' and F'' are invalid. By the induction hypothesis, these tests are in $\Pi_{k-1}CSYMLOG$. Thus the symmetric complementing game can be implemented by a symmetric complementing machine with complement bound k and space bound $O(\log n)$. By Lemma 5.2, the player wins iff F is invalid. We have thus shown that $\Sigma_k QBF\oplus$ invalidity is in $\Sigma_k CSYMLOG$.

Now let M be a symmetric complementing machine with complement bound k and space bound $\log n$. Let $\vdash_s$ be the noncomplement moves of M. For each k', $1 \leq k' \leq k$, let $\mathscr{I}_{k'}$ be the set of configurations of M with $\leq \log n$ nonblank cells per work tape and for which there is a complement bound of k'. Let $\mathscr{C}_{k'}$ be the configurations of $\mathscr{I}_{k'}$ which have a complement as a next move. By the induction hypothesis we can assume for each $I \in \mathscr{C}_{k-1}$ a formula $F'(I)$ of $\Pi_{k-1}\text{QBF}\oplus$ such that $F'(I)$ is invalid iff $\text{OUTCOME}(I') = \underline{\text{false}}$ for each complement move (I,I') from I.

For each $I \in \mathscr{I}_k$ we assume a distinct variable x_I. Let $X = \{x_I \mid I \in \mathscr{I}_k\}$. Let W' be configurations of M which are accepting and have $\leq \log n$ nonblank cells per work tape. For each $I \in W'$, let g_I be the formula $x_I \oplus \underline{\text{true}}$. Thus g_I is $\underline{\text{true}}$ iff $x_I = \underline{\text{false}}$. For each $I \in \mathscr{C}_{k-1}$, let g_I be the formula $(\neg x_I) \vee F'(I)$. For each $I \in \mathscr{I}_k$ and $J \in W' \cup \mathscr{C}_{k-1}$, let $f_{I,J} = (\underline{\text{true}} \oplus \neg x_I) \wedge (\bigwedge_{I_1 \vdash_{s,k} I_2} x_{I_1} \oplus \neg x_{I_2}) \wedge g_J$ where $\vdash_{s,k} = \vdash_s \cap (\mathscr{I}_k \times \mathscr{I}_k)$. Thus $(\exists X) f_{I,J}$ is invalid iff there exists a computation sequence $I_1, \ldots, I_j$ such that $I_1 = I$, $I_j = J$ and $\text{OUTCOME}(J) = \underline{\text{true}}$.

Now for each $I \in \mathscr{I}_k$ and $J \in W' \cup \mathscr{C}_{k-1}$ let x_I^J be a new distinct variable, and let $f'_{I,J}$ be derived from $f_{I,J}$ by substituting $x_{I'}^J$ for each instance of variable $x_{I'}$, for each $I' \in \mathscr{I}_k$. Let $X' = \{x_I^J \mid I \in \mathscr{I}_k \text{ and } J \in W' \cup \mathscr{C}_{k-1}\}$. For each $I \in \mathscr{I}_k$, let $F(I) = (\exists X') \bigwedge_{J \in W' \cup \mathscr{C}_{k-1}} f'_{I,J}$. Clearly $F(I)$ is in $\Sigma_k\text{QBF}\oplus$ and furthermore, $F(I)$ is invalid iff $\text{OUTCOME}(I) = \underline{\text{true}}$. Hence $F(I_0(\omega))$ is invalid iff M accepts ω.

We have thus shown that the invalidity problem for Σ_kQBF$\oplus$ is complete in Σ_kCSYMLOG. By Propositions 5.1 and 2.1, the invalidity problem for Π_kQBF$\oplus$ is complete in Π_kCSYMLOG. □

5.3 k-Connectivity

Given a graph $G = (V,E)$ and vertices $u,v \in V$, let k-PATHS(G,u,v) be the problem: are there k vertex disjoint paths from u to v? The problem 1-PATHS is commonly called the UGAP problem.

THEOREM 5.3. UGAP is complete in NSYMLOG.

Proof [Lewis and Papadimitriou, 80]. Given an undirected graph G of n vertices with distinguished vertices u,v, we nondeterministically traverse a path in G from u, and accept if the vertex v is reached. This can easily be done by a nondeterministic machine in space $O(\log n)$ to store the currently visited vertex. But this nondeterministic machine can be made symmetric since any edge can be traversed in both directions.

On the other hand, suppose M is a nondeterministic symmetric machine with $\log n$ space boundly and input string $\omega \in \Sigma^n$. Let $\mathcal{I}$ be the configurations of M, with space $\leq \log n$, let $\vdash_s \subseteq \mathcal{I} \times \mathcal{I}$ be the nondeterministic moves of M, and let $W \subseteq \mathcal{I}$ be the accepting configurations. We construct a undirected graph with vertices $V = \mathcal{I} \cup \{I_f\}$ where $I_f \notin \mathcal{I}$, and edges $E = \{\{I,I'\} | I \vdash_s I'\} \cup \{\{I,I_f\} | I \vdash I' \text{ for some } I' \in W\}$. Then M accepts ω iff there is a path in (V,E) from $I_0(\omega)$ to I_f. □

By Theorems 2.1 and 5.3,

COROLLARY 5.3. The complement of the UGAP problem is complete in Π_1CSYMLOG.

THEOREM 5.4. For each $k \geq 1$, k-PATHS is complete in NSYMLOG.

We also show that for each $k \geq 0$, the k-connected components recognition problem for an undirected graph is in NSYMLOG.

By Menger's Theorem [Bondy and Murty, 77], for any graph $G = (V,E)$ and vertices $u, v \in V$, $k\text{-PATHS}(G,u,v) \leftrightarrow (\forall x_1,\ldots,x_{k-1} \in V - \{u,v\})\text{UGAP}(G',u,v)$ where G' is derived from G by deleting vertices $x_1,\ldots,x_{k-1}$ and all edges connected to these vertices. Thus we have a deterministic log space reduction to UGAP, which by Theorem 5.3 is in NSYMLOG. Theorem 5.4 is thus proved.

Let a graph $G = (V,E)$ be k-*connected* if for all distinct vertices $u, v \in V$, there exists k vertex disjoint paths from u to v. We define a k-*connected component* of G to be a maximal k-connected subgraph of G. (Note: to facilitate planarity testing, [McLane, 37] and others define "tri-connected" components somewhat differently. However McLane's components are homeomorphic (i.e., derived by replacing paths, with internal vertices of valence 2, by edges) to the 3-connected components of our definition.)

By Menger's Theorem, any two k-connected components intersect at no more than $k-1$ vertices. Thus some k vertices suffice to uniquely determine any k-connected component of G. Let $k\text{-CC}(G,x,\{v_1,\ldots,v_k\})$ be the problem: is vertex x in the k-connected component of G determined by $\{v_1,\ldots,v_k\}$?

COROLLARY 5.4. k - CC is complete in NSYMLOG.

Proof. By Menger's Theorem,

$$k\text{-CC}(G,x,\{v_1,\ldots,v_k\}) \leftrightarrow \bigwedge_{1\leq i\leq k} k\text{-PATHS}(G,x,v_i) .$$

Thus we may apply Theorem 5.4 □

5.4 Minimum Spanning Forests

Here we show the Π_1CSYMLOG contains the problem of recognizing an edge of a (unique) minimum spanning forest.

Let $G = (V,E)$ be an undirected graph with a mapping $W : E \to \mathbb{N}^+$ labeling the edges with distinct positive integers. Consider the following well known greedy algorithm for constructing a minimum (weight) spanning forest of G:

Input graph $G = (V,E)$ and edge weighting W.

[1] sort the edges $E = \{e_1,\ldots,e_m\}$ so that $W(e_i) < W(e_{i+1})$ for $i = 1,\ldots,m-1$.

[2] $SF \leftarrow \emptyset$

[3] for $i = 1$ to m do

if $SF \cup \{e_i\}$ contains no cycles then $SF \leftarrow SF \cup \{e_i\}$

Return (V,SF).

Note that the minimum spanning forest output by this algorithm is *unique* for a fixed W (even though in general there may exist many spanning forests of a given graph).

Let SPANNING-EDGE(G,W,e) be *true* if $e \in SF$ and *false* otherwise.

THEOREM 5.5. SPANNING-EDGE is complete in Π_1CSYMLOG

Proof. Let $e = \{u,v\}$ be an edge of $G = (V,E)$ and let $G_e = (V, \{e' \in E \mid W(e') < W(e)\})$. Then SPANNING-EDGE$(G,W,e) \leftrightarrow \neg$UGAP$(G_e,u,v)$. The result then follows from Corollary 5.3. □

5.5. Other Graph Recognition Problems Contained in Π_1CSYMLOG

Here we note that the recognition problems for many interesting and commonly found classes of graphs (including chordal graphs, comparability graphs, interval graphs, split graphs, and permutation graphs) are contained in Π_1ASYMLOG. Our proofs use known characterization Lemmas.

Let $G = (V,E)$ be an undirected graph. Let its *complement* be $\bar{G} = (V, \{\{u,v\} \notin E | u,v \in V\})$. We define here some graphs commonly found in the literature. Each has a characterization Lemma which immediately implies by Corollary 5.3 its recognition problem is in Π_1CSYMLOG (by a deterministic logspace reduction to the complement of the UGAP problem). G is a *chordal graph* if every cycle C of length >3 contains a *chord* (an edge connecting two nonconsecutive vertices of C).

LEMMA 5.2. G is chordal iff for every vertex $v \in V$ and cycle C of length >3, if C contains v then C has a chord $\{x,y\}$ such that both x and y are of distance ≤ 2 from v.

Proof. Repeatedly apply the chordal graph definition. □

G is a *comparability graph* if its edges may be transitively directed.

LEMMA 5.3. [Gilmore and Hoffman, 64] G is a comparability graph iff for every cycle C of G, if $\{x,y\} \notin E$ for every pair of vertices x, y of distance 2 in C, then C has an even number of edges.

G is an *interval graph* if its vertices can be put into 1-1 correspondence with a set of intervals on the real line, such that two vertices are connected by an edge of G iff their corresponding intervals have nonempty intersection.

LEMMA 5.4. [Gilmore and Hoffman, 64] G is an interval graph iff G is a chordal graph and $\bar{G}$ is a comparability graph.

G is a *split graph* if its vertex set V can be partitioned into sets V_1, V_2 such that $E(V_1) = \emptyset$ and $(V_2, E(V_2))$ is a complete graph.

LEMMA 5.5. [Földes and Hammer, 1977] G is a split graph iff G and $\bar{G}$ are chordal graphs.

$G = (V,E)$ is a *permutation graph* if $V = \{v_1, \ldots, v_n\}$ and there is a permutation σ of $\{1, \ldots, n\}$ such that $\{v_i, v_j\} \in E$ iff $(i-j)(\sigma^{-1}(i) - \sigma^{-1}(j)) < 0$.

LEMMA 5.6. [Pnueli, Lempel, and Even, 71] G is a permutation graph iff both G and $\bar{G}$ are comparability graphs.

By the above Lemmas and Corollary 5.3,

THEOREMS 5.7-11. The recognition problem for each of the graph classes: chordal graphs, comparability graphs, interval graphs, split graphs, permutation graphs are in Π_1CSYMLOG.

G is *bipartite* if the vertex set V may be partitioned into disjoint sets V_1, V_2 such that $E \subseteq \{\{u,v\} \mid u \in V_1, v \in V_2\}$.

LEMMA 5.7. G is bipartite iff G has no cycle of odd length.

By using this characterization Lemma, [Jones, Lien, and Laaser, 76] show the recognition problem for nonbipartite graphs is $\leq_{log}$ equivalent to the complement of UGAP. Thus, by Corollary 5.3,

THEOREM 5.12. The bipartite graph recognition problem is complete in Π_1CSYMLOG.

Also, [Jones, Lien, and Laaser, 76] give restricted cases of the NP-complete problems CHROMATIC NUMBER, CLIQUE COVER, EXACT COVER, HITTING SET, and show their restricted problems are $\leq_{log}$ equivalent to UGAP, and thus complete in NSYMLOG

5.6 Valence 3 Planarity Testing is in Π_3CSYMLOG

5.6.a. Embedding Rotations

Let $G = (V,E)$ be a undirected graph with vertex set V and undirected edge set $E \subseteq \{\{u,v\} | \text{distinct } u, v \in V\}$. Let $D(E) = \{(u,v) | \{u,v\} \in E\} \cup \{(v,u) | \{u,v\} \in E\}$ be the set obtained by directing edges of E. Following [Edmonds, 60] (also see [White, 73]) we define an *embedding* onto an oriented surface purely combinatorially; let an *embedding rotation* be a set $\theta = \{\theta_v | v \in V\}$ where θ_v is a cyclic permutation of the directed edges $D_v(E) = \{(x,y) \in D(E) | x=v\}$ of $D(E)$ departing from vertex v. Intuitively, θ_v gives the clockwise rotation of edges as they are embedded around vertex v, in a graph with a planar embedding.

Let $R(\theta) \subseteq D(E) \times D(E)$ be the relation such that $e_1^- R(\theta) e_2$ iff directed edge e_1 departs from the same vertex v that directed edge e_2 departs from, and e_2 appears immediately after e_1 in θ_v (where e_1^- is the reverse of edge e_1). See Figure 1.

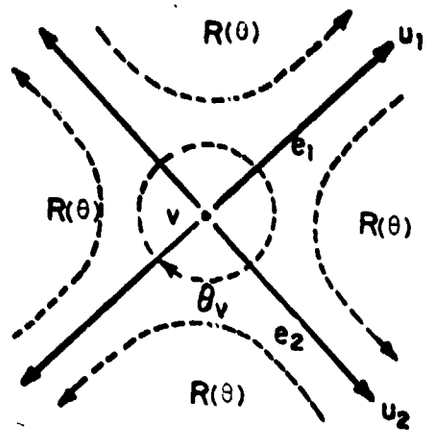


Figure 1.

$R(\theta)$ is easily shown to be partitioned into a set of cyclic permutations. Let c be the number of 1-connected components of G. Let $\lambda(\theta)$ be the number of orbits of $R(\theta)$. By Euler's formula, the embedding rotation θ is *planar* if $\lambda(\theta) - |E| + |V| = 2c$. Intuitively, if θ is planar and $c = 1$ then the orbits of $R(\theta)$ are in 1-1 correspondence with the connected regions of the embedding, with the borders of the regions oriented counter-clockwise. If θ is planar and $c \geq 1$, then c orbits of R will be associated with the exterior regions of the embedding.

By Edmonds' [60] characterization, the graph G is *planar* iff it has a planar embedding rotation.

5.6.b. The Basis Cycles

Let us impose an arbitrary, fixed numbering of the edges E from $1,\ldots,|E|$. We consider this numbering to be an edge weighting of the 2-connected graph $G = (V,E)$. The greedy algorithm of Section 5.4 constructs a *unique* minimum spanning forest (V,SF_G). For each directed edge (u,v),

where $\{u,v\} \in E - SF_G$, there is a unique directed simple cycle $C_{(u,v)}$ containing (u,v) and directed edges derived from SF_G. We call $C_{(u,v)}$ a *basis cycle*. Let $\mathcal{C}$ be the set of all basis cycles. By applying Theorem 5.5 we have:

LEMMA 5.8. Π_1CSYMLOG contains the test: Is edge sequence C in $\mathcal{C}$?

5.6.c. The Bridges of G

Let C be a basis cycle in $\mathcal{C}$ of graph $G=(V,E)$. Let a *bridge* of C be a maximal edge set $B \subseteq E - \{\{u,v\} | (u,v) \in C\}$ such that $\forall e_1, e_2 \in B$, there is a path p in B containing both e_1 and e_2, but visiting vertices of C only at the endpoints of p. Given C, we can construct in deterministic log-space a graph (this graph is derived from G by substituting a distinct new vertex v_e and edge $\{u,v_e\}$ for each edge $e=\{u,v\}$ of E such that a vertex v appears in C) whose 1-connected components are in 1-1 correspondence with the bridges of C. Thus, by Lemma 5.8 and Corollary 5.4,

LEMMA 5.9. The bridges of all the cycles of the basic $\mathcal{C}$ may be recognized in Σ_2CSYMLOG.

5.6.d. Embedding Formulas for Graphs of Valence 3

Suppose we have an oracle for Σ_2CSYMLOG. Given this oracle, we provide a deterministic log space construction of a CNF$\oplus$ formula F_G which will encode embedding rotations of graph $G=(V,E)$ of valence 3.

For each vertex $v \in V$ and directed edges $e_1, e_2, e_3 \in D_v(E)$ departing from v, we have a distinct boolean variable $O_v(e_1,e_2,e_3)$. Let h_1 be the CNF$\oplus$ formula:

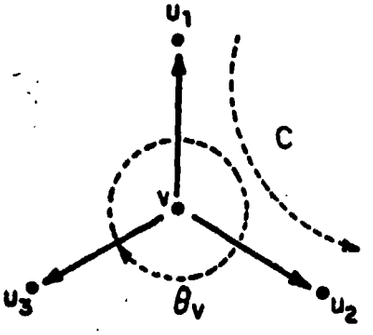
$$\bigwedge_{\substack{v \in V \\ e_1, e_2, e_3 \in D_v(E)}} (O_v(e_1,e_2,e_3) \oplus O_v(e_3,e_2,e_1)) \wedge (O_v(e_1,e_2,e_3) \oplus \neg O_v(e_2,e_3,e_1))$$

Note that h_1 holds just for truth assignments to O_v where there exists an embedding rotation $\theta = \{\theta_v \mid v \in V\}$ where $\forall v \in V$ and $e_1, e_2, e_3 \in D_v(E)$ departing from v, $O_v(e_1,e_2,e_3) =$ <u>true</u> iff e_2 is cyclically ordered between e_1 and e_3 in θ_v.

Also, for each undirected edge $e \in E$ and basis cycle $C \in \mathscr{C}$ not containing e, we have a boolean variable $\mathrm{OUT}(C,e)$. (Intuitively, this variable will be <u>true</u> iff e is embedded into the exterior of the closed region of the sphere defined by C.) Let h_2 be the CNF$\oplus$ formula:

$$\bigwedge_{\substack{C \in \mathscr{C} \\ (u_1,v),(v,u_2) \in C \\ \{v,u_3\} \in E \\ \text{distinct } v,u_1,u_2,u_3 \in V}} O_v((v,u_1),(v,u_2),(v,u_3)) \oplus \neg\, \mathrm{OUT}(C,\{v,u_3\})$$

Note that $h_1 \wedge h_2$ holds just when an embedding rotation θ induced such that for each $v \in V$ and basis cycle $C \in \mathscr{C}$ containing vertex v, if $(u_1,v),(v,u_2)$ are directed edges of C and $\{v,u_3\}$ is any other edge containing v, then $\{v,u_3\}$ is embedded to the exterior of the region defined by C if θ_v cyclically orders (v,u_2) between (v,u_1) and (v,u_3) (see Figure 2.)



<u>Figure 2</u>.

By Lemma 5.8, h_2 can be constructed in deterministic log-space, given a Π_1CSYMLOG oracle. Finally, let h_3 be the formula

$$\bigwedge_{\substack{C\in\mathscr{C} \\ e_1,e_2\in E \\ e_1,e_2 \text{ are in the} \\ \text{same bridge of } C}} \mathrm{OUT}(C,e_1) \oplus \neg\, \mathrm{OUT}(C,e_2)$$

Note that h_3 holds just when for each bridge B of any cycle $C\in\mathscr{C}$, either all edges of B are embedded interior to the region defined by C, or all edges of B are embedded exterior to the region defined by C. By Lemmas 5.8 and 5.9, h_3 may be constructed in deterministic log-space, given an oracle for Σ_2CSYMLOG.

Let $H_G = h_1 \wedge h_2 \wedge h_3$.

LEMMA 5.10. A graph G of valence 3 is planar iff H_G is satisfiable.

Proof. Suppose G is planar. Then by Edmond's characterization, G has some planar embedding orientation θ. We use θ to define the truth values for the O_v and OUT variables. They clearly satisfy h_1, h_2, and h_3. Thus we can satisfy H_G.

On the other hand, suppose H_G is satisfiable. Let θ be the embedding rotation induced by the O_v variables. Let $\lambda(\theta)$ be the number of orbits of $R(\theta)$. Now we must show $\lambda(\theta) - |E| + |V| = 2$ implying by Euler's formula θ is planar. Fix the spanning forest $SF = SF_G$. Suppose we delete

an edge $e \in E - SF$ from G, so that the resulting graph is $G' = (V,E')$ with $E' = E - \{e\}$. We claim that if θ' is the resulting embedding rotation, then $\lambda(\theta) - |E| + |V| = \lambda(\theta') - |E'| + |V|$. Clearly $|E'| = |E| - 1$ so we must show $\lambda(\theta') = \lambda(\theta) - 1$.

Let $e = \{u,v\}$ be the undirected edge to be deleted from $E - SF$. Let π_1, π_2 be the orbits of $R(\theta)$ containing directed edges $d_1 = (u,v)$, $d_2 = (v,u)$ respectively, so $d_1^- = d_2$ and $d_2^- = d_1$. We claim now that $\pi_1 \neq \pi_2$.

For each $i \in \{1,2\}$, let $C_{d_i} \in \mathscr{C}$ be the unique basis cycle which contains directed edge d_i. If π_i is a cyclic permutation of C_{d_i} then C_{d_i} and π_i do not contain the reverse edge d_i^-, so $\pi_1 \neq \pi_2$.

Otherwise, for each $i = 1,2$ let (x_i,y_i) be the first edge of π_i following d_i such that (x_i,y_i) is not in C_{d_i}. Let f_i be the directed edge of π_i immediately preceding (x_i,y_i). By definition of orbits, (x_i,y_i) immediately follows f_i^- in θ_{x_i}. Note also that f_i must be a directed edge in C_{d_i}. Let g_i be the directed edge of C_{d_i} immediately following f_i. Then (x_i,y_i) is cyclicly ordered between f_i^- and g_i in θ_{x_i}. Since h_2 is satisfied, $OUT(C_{d_i}, \{x_i,y_i\}) = \underline{\text{false}}$ for $i = 1,2$. But since C_{d_2} is the reverse of directed cycle C_{d_1}, g_2^- appears just before f_2^- in C_{d_1} and so $OUT(C_{d_1}, \{x_2,y_2\}) = \underline{\text{true}}$.

However, we claim $OUT(C_{d_1}, \{x,y\}) = \underline{\text{false}}$ for all $\{x,y\} \in E$ such that (x,y) is contained in π_1 but not C_{d_1}. This has already been proved for $x = x_1$ and $y = y_1$. For any subsequence π' of π_1 contained on only a single bridge of C_{d_1}, since h_3 is satisfied we have $OUT(C_{d_1}, \{x,y\}) = OUT(C_{d_1}, \{x',y'\})$ for all (x,y) and (x',y') in π'. Also, if $(x,y)\pi''(x',y')$ is a subsequence of π_1 where π'' is a

is a subsequence of C_{d_1} but (x,y) and (x',y') do not appear in C_{d_1}, then since h_2 is satisfied we have $OUT(C_{d_1},\{x,y\}) = OUT(C_{d_1},\{x',y'\})$.

If (x_2,y_2) is in π_1, then we have just shown $OUT(C_{d_1},\{x_2,y_2\}) = \underline{false}$, contradicting our previous proof then $OUT(C_d, \{x_2,y_2\}) = \underline{true}$. Hence (x_2,y_2) is not in π_1 but is by assumption in π_2, so $\pi_1 \neq \pi_2$.

Without loss of generality, let $\pi_1 = d_1\pi_1'$ and $\pi_2 = d_2\pi_2'$ where neither π_1' nor π_2' contain d_1 or d_2. Hence when e is deleted from $E - SF$, the orbits π_1 and π_2 of $R(\theta)$ are merged into a single new orbit $\pi_1'\pi_2'$ of $R(\theta')$, and no other orbits are modified.

Thus $\lambda(\theta') = \lambda(\theta) - 1$, and we have shown $\lambda(\theta) - |E| + |V|$ remains invariant. We repeat this process (deleting edges not in SF) until we have only $E' = SF$. Let c be the number of maximal trees in the spanning forest SF, c is also a number of 1-connected components of SF. Then $\lambda(\theta') = c$, $|SF| = n-c$ and $|V| = n$. Hence our invariant is $\lambda(\theta) - |E| + |V| = \lambda(\theta') - |SF| + |V| = 2c$, which implies by Euler's formula the embedding orientation θ is planar. Thus, by Edmonds characterization, G is planar. □

We have by applying Theorem 5.2 and Lemma 5.10,

THEOREM 5.13. Planarity testing of valence 3 graphs is in Π_3CSYMLOG.

6. CONCLUSION

It may be significant that the symmetric complementing machine introduced in this paper has applications to many combinatorial problems found in practice, such as spanning trees, k-connectivity and planarity testing. In this case the theoretical study of a new machine type led us to the discovery of new techniques for practical combinatorial algorithm design. For example, applying our probabilistic decision algorithm for symmetric games to our proof that valence 3 planarity testing is in Π_3CSYMLOG, yields a new probabilistic algorithm for planarity testing; this algorithm has a quite different structure than any previously known deterministic planarity testing algorithm such as that of [Hopcroft and Tarjan, 74]. This indicates to us that the field of combinatorial algorithms, as well as the field of abstract computational complexity would benefit by further study of unusual machine types and their decision algorithms.

Preliminary Work and Acknowledgments

In a preliminary draft of this paper, we utilized a restricted type of alternating machine whose nonalternation next moves are a symmetric relation. Dexter Kozen pointed out to us that such symmetric alternation machines are too restricted for our intended applications (in particular, thy do not satisfy a complementation property such as Proposition 2.1).

A succeeding draft of this paper generalized this machine to allow nonalternation moves to be essentially a cross-product of a symmetric and a deterministic relation. Joe Halpern informed us of certain difficulties of this generalization.

The symmetric complementing machine described in this current draft satisfies the required complementation property of Proposition 2.1 and also has efficient decision algorithms if the machine is both space and complementation bounded. Michael Sipser has also suggested an equivalent machine; this is a symmetric alternating machine M as we originally defined (with symmetric nonalternation moves), but with a modified definition of acceptance:

M *accepts* (*rejects*, respectively) from an existential (universal, respectively) configuration I if there exists a finite computation sequence $I = I_1,\ldots,I_{j-1},I_j$ where $I_1,\ldots,I_{j-1}$ are existential (universal, respectively) and I contains an accepting (rejecting, respectively) state or I_j is universal (existential, respectively) and M accepts (rejects, respectively) from I_j. Also, M *rejects* (*accepts*, respectively) from existential (universal, respectively) configuration I iff M does not accept (does not reject) from I. Thus Sipser's definitions for acceptance and rejection of these machines are duals. This is the same as the standard definition of acceptance of an alternating machine from an existential configuration, but differs from the standard definition of acceptance from a universal configuration so as to allow for complementations of languages.

The author wishes to thank Larry Denenberg, Vassos Hadzilacos, Joe Halpern, Harry Lewis, A. Prasad, Michael Sipser, and Paul Spirakis for a careful reading and many useful comments on preliminary drafts of this paper.

REFERENCES

Adleman, L., "Two theorems on random polynomial time," *18th IEEE Symposium on Foundations of Computer Science*, 75-83 (1978).

Aleliunas, R., R.M. Karp, R.H. Lipton, L. Lovasz and C. Rackoff, "Random walks, universal traversal sequences, and complexity of maze problems," *Proc. 20th Annual Symposium on Foundations of Computer Science*, 218-223 (1979).

Bondy, J.A. and U.S.R. Murty, *Graph Theory with Applications*, American Elsevier, NY (1977).

Chandra, A.K., D.C. Kozen and L.J. Stockmeyer, "Alternation," *J. ACM*, 1981.

Cook, S.A., "Towards a complexity theory of synchronous parallel computation," Presented at *Internationales Symposium über Logik und Algorithmik zu Ehren von Professor Hort Specker*, Zürich, Switzerland, February 1980.

Csanky, L., "Fast Parallel Matrix Inversion Algorithms," *SIAM J. Comput.* 5, 618-623 (1976).

Dymond, P. and S.A. Cook, "Hardware complexity and parallel computation," *IEEE FOCS Conference*, 1980.

Edmonds,J., "A combinatorial representation for polyhedral surfaces," *Amer. Math. Soc. Notices* 7, 646 (1960).

Even, S., A. Pnueli and A. Lempel, "Permutation graphs and transitive graphs," *J. Assoc. Comput. Mach.* 19, 400-410, MR47 #1675 (1972).

Foldes, S. and P.L. Hammer, "Split graphs," *8th Southeastern Conf. on Combinatorics, Graph Theory and Computing* (F. Hoffman et al., eds.), Louisiana State Univ., Baton Rouge, Louisiana, 311-315 (1977).

Fortune, S. and J. Wyllie, "Parallelism in random access machines," In *Proc. of the 10th ACM Symposium on Theory of Computation*, 114-118 (1978).

Gill, J., "Complexity of probabilistic Turing machines," *SIAM J. of Computing*, 6(4), 675-695 (1977).

Gilmore, P.C. and A.J. Hoffman, "A characterization of comparability graphs and of interval graphs," *Canad. J. Math.* 16, 539-548 MR31 #87 (1964).

Goldschlager, L., "A unified approach to models of synchronous parallel machines," In *Proc. 10th Annual ACM Symposium on the Theory of Computing*, San Diego, California, 89-94 (1978).

Hirschberg, D.C., "Parallel algorithms for the transitive closure and the connected components problems," In *Proc. 8th Annual ACM Symposium on the Theory of Computing*, 55-57 (1976).

Hopcroft, J.E. and R.E. Tarjan, "Efficient planarity testing," *J. ACM*, 21 549-568 (1974).

Hopcroft, J.E. and R.E. Tarjan, "Dividing a graph into triconnected components," *SIAM J. Computing* 2:3 (1973b).

Ja'Ja', J., and J. Simon, "Some space-efficient algorithms," *Proc. 17th Allerton Conference*, pp. 677-684, also Technical Report CS 80-23, Dept. Comp. Science, Penns. State Univ., Sept. 1980.

Ja'Ja', J. and J. Simon, "Parallel algorithms in graph theory: Planarity testing," Techn. Report CS-80-14, Penns. State Univ., June 1980.

Jones, N.D., Y.E. Lien, and W.T. Leaser, "New problems complete for nondeterministic log space," *Math. System Theory* 10, pp. 1-17 (1976).

Karp, R.M., and R.J. Lipton, "Some connections between nonuniform and uniform complexity classes," *Proc. 17th Annual ACM Symposium on Theory of Computing*, April, 1980, 302-309. (Also presented at the Specker Symposium on Complexity, Zürich, February, 1980).

Lewis, H.R. and C.H. Papadimitriou, "Symmetric space bounded computation," *ICALP80*.

McLane, S., "A combinatorial condition for planar graphs," *Fundamenta Math.* 28, 22-32 (1937).

McLane, S., "A structural characterization of planar combinatorial graphs," *Duke Math. J.* 3, 46-472, (1937).

Pnueli, A., A. Lempel, and S. Even, "Transitive orientation of graphs and identification of permutation graphs," *Canad. J. Math*, 23, 160-175 (1971).

Rabin, M.O., "Probabilistic algorithms," *Algorithms and Complexity, New Directions and Recent Results*, edited by J. Traub, Academic Press, 1974.

Reif, J.H., "The computational complexity of probabilistic parallel machines," to appear.

Savitch, W. J. and M. [illegible]son, "Time random access machines with parallel processing," J. *ACM*, 26, 108-118 (1979).

Schonhage, A., "Storage modification machines," Technical Report, Mathematisches Institut, Universität Tübingen, Germany, 1979.

Tarjan, R.E., "Depth [illegible] search and linear graph algorithms," *SIAM J. Computing* 1:2, 146-160 (1972).

White, A., *Graphs, Groups and Surfaces*, North-Holland Publishing Co., American Elsevier Co., New York, New York, 1973.

Wyllie, J.C., "The complexity of parallel computations," Ph.D. Thesis and TR-79-387, Dept. of Computer Science, Cornell University, 1979.